Name: ________________

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Math 2D 8AM Final Exam
Question Page

For the following questions, you should show your work if you want credit. Answer in the spaces provided on the following pages and mark your answer clearly. You are allowed a scientific calculator but nothing else.

1. (10 points) Find the center of mass of the lamina occupying the rectangle $R = [0, 2] \times [0, 1]$ and having density function
$$\rho(x, y) = xy.$$
2. (10 points) Find the **surface area** of the portion of the sphere
$$x^2 + y^2 + z^2 = 9$$
above the plane $z = 1$.
3. (10 points) Show that the volume inside the sphere of radius 1 is equal to $\frac{4}{3}\pi$ by evaluating a triple integral in spherical coordinates.
4. (10 points) Set up but **do not evaluate** an iterated triple integral for the volume of the region in the first octant enclosed by the coordinate planes and the plane $2x+3x+4z = 4$.
5. (10 points) Consider the following double integral:
$$\iint_R (x - y)(y + 2x)dA$$
where R is the region bounded the lines
$$y - x = 0, \quad y - x = 1, \quad y + 2x = 0, \quad y + 2x = 1.$$
Rewrite this double integral using the new variables:
$$u = y - x \quad v = y + 2x.$$
I'll even give you x and y in terms of u and v:
$$x = \frac{v - u}{3} \qquad y = \frac{2u + v}{3}.$$
Don't bother computing the integral's numerical value.

6. (10 points) Find an equation of the tangent plane to the graph of

$$f(x, y) = 2x^2 + 3xy - 3y^3$$

at the point $(1, 2, -16)$.

7. (10 points) Find an equation of the plane passing through the three points

$$(1, 1, 1) \qquad (1, 3, -1) \qquad (2, 1, 0)$$

8. (10 points) Describe two important geometric properties possessed by the gradient of a function of two variables.

9. (10 points) Find the absolute maximum value of $f(x, y) = x + 3y$ on the ellipse

$$2x^2 + 3y^2 = 1.$$

10. (10 points) What is your favorite thing about multivariable calculus (excluding your professor)?

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For the following questions, you should show your work if you want credit. Answer in the spaces provided on the following pages and mark your answer clearly. You are allowed a scientific calculator but nothing else.

1. (10 points) Find the center of mass of the lamina occupying the rectangle $R = [1,3] \times [0,1]$ and having density function
$$\rho(x, y) = xy.$$
2. (10 points) Find the **surface area** of the portion of the sphere
$$x^2 + y^2 + z^2 = 4$$
above the plane $z = 1$.
3. (10 points) Show that the volume inside the sphere of radius 1 is equal to $\frac{4}{3}\pi$ by evaluating a triple integral in spherical coordinates.
4. (10 points) Set up but **do not evaluate** an iterated triple integral for the volume of the region in the first octant enclosed by the coordinate planes and the plane $x+2y+3z = 3$.
5. (10 points) Consider the following double integral:
$$\iint_R (x - y)(y + 3x) dA$$
where R is the region bounded the lines
$$y - x = 0, \quad y - x = 2, \quad y + 3x = 0, \quad y + 3x = 1.$$
Rewrite this double integral using the new variables:
$$u = y - x \quad v = y + 3x.$$
I'll even give you x and y in terms of u and v:
$$x = \frac{v - u}{4} \qquad y = \frac{3u + v}{4}.$$
Don't bother computing the integral's numerical value.

6. (10 points) Find an equation of the tangent plane to the graph of

$$f(x, y) = 2x^2 + 3xy - 3y^3$$

at the point $(1, 1, 2)$.

7. (10 points) Find an equation of the plane passing through the three points

$$(1, 1, 1) \qquad (1, 3, -1) \qquad (-2, 1, 0)$$

8. (10 points) Describe two important geometric properties possessed by the gradient of a function of two variables.

9. (10 points) Find the absolute maximum value of $f(x, y) = x - 3y$ on the ellipse

$$2x^2 + 3y^2 = 1.$$

10. (10 points) What is your favorite thing about multivariable calculus (excluding your professor)?

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$$\rho(x, y) = xy.$$
2. (10 points) Find the **surface area** of the portion of the sphere
$$x^2 + y^2 + z^2 = 10$$
above the plane $z = 1$.
3. (10 points) Show that the volume inside the sphere of radius 1 is equal to $\frac{4}{3}\pi$ by evaluating a triple integral in spherical coordinates.
4. (10 points) Set up but **do not evaluate** an iterated triple integral for the volume of the region in the first octant enclosed by the coordinate planes and the plane $3x+2y+3z = 3$.
5. (10 points) Consider the following double integral:
$$\iint_R (x - y)(y + 2x)dA$$
where R is the region bounded the lines
$$y - x = 0, \quad y - x = 3, \quad y + 2x = 0, \quad y + 2x = 1.$$
Rewrite this double integral using the new variables:
$$u = y - x \quad v = y + 2x.$$
I'll even give you x and y in terms of u and v:
$$x = \frac{v - u}{3} \qquad y = \frac{2u + v}{3}.$$
Don't bother computing the integral's numerical value.

6. (10 points) Find an equation of the tangent plane to the graph of

$$f(x, y) = 2x^2 + 3xy - 3y^3$$

at the point $(1, -1, 2)$.

7. (10 points) Find an equation of the plane passing through the three points

$$(1, 1, 1) \qquad (1, 3, -1) \qquad (2, 2, 1)$$

8. (10 points) Describe two important geometric properties possessed by the gradient of a function of two variables.

9. (10 points) Find the absolute maximum value of $f(x, y) = 2x + 3y$ on the ellipse

$$2x^2 + 3y^2 = 1.$$

10. (10 points) What is your favorite thing about multivariable calculus (excluding your professor)?

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For the following questions, you should show your work if you want credit. Answer in the spaces provided on the following pages and mark your answer clearly. You are allowed a scientific calculator but nothing else.

1. (10 points) Find the center of mass of the lamina occupying the rectangle $R = [0,2] \times [0,2]$ and having density function
$$\rho(x,y) = xy.$$

2. (10 points) Find the **surface area** of the portion of the sphere
$$x^2 + y^2 + z^2 = 5$$
above the plane $z = 1$.

3. (10 points) Show that the volume inside the sphere of radius 1 is equal to $\frac{4}{3}\pi$ by evaluating a triple integral in spherical coordinates.

4. (10 points) Set up but **do not evaluate** an iterated triple integral for the volume of the region in the first octant enclosed by the coordinate planes and the plane $2x+y+3z = 3$.

5. (10 points) Consider the following double integral:
$$\iint_R (x-y)(y+3x)dA$$
where R is the region bounded the lines
$$y - x = 0, \quad y - x = 4, \quad y + 3x = 0, \quad y + 3x = 1.$$
Rewrite this double integral using the new variables:
$$u = y - x \quad v = y + 3x.$$
I'll even give you x and y in terms of u and v:
$$x = \frac{v-u}{4} \qquad y = \frac{3u+v}{4}.$$
Don't bother computing the integral's numerical value.

6. (10 points) Find an equation of the tangent plane to the graph of

$$f(x, y) = 2x^2 + 3xy - 3y^3$$

at the point $(2, 1, 11)$.

7. (10 points) Find an equation of the plane passing through the three points

$$(1, 1, 1) \qquad (1, 3, -1) \qquad (2, 1, 2)$$

8. (10 points) Describe two important geometric properties possessed by the gradient of a function of two variables.

9. (10 points) Find the absolute maximum value of $f(x, y) = 3x + 2y$ on the ellipse

$$2x^2 + 3y^2 = 1.$$

10. (10 points) What is your favorite thing about multivariable calculus (excluding your professor)?